2.12 INTRODUCTION TO ROBOTICS
BRIEF - DETERMINANTS

(by Alberto Rodriguez)

→ Algebraic definition

$$\det \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = a_{11} \cdot a_{22} - a_{12} \cdot a_{21}$$

$$\det \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{aligned} & a_{11} \cdot a_{22} \cdot a_{33} \\ & + a_{21} \cdot a_{32} \cdot a_{13} \\ & + a_{12} \cdot a_{23} \cdot a_{31} \\ & - a_{13} \cdot a_{22} \cdot a_{31} \\ & - a_{12} \cdot a_{21} \cdot a_{33} \\ & - a_{23} \cdot a_{32} \cdot a_{11} \end{aligned}$$

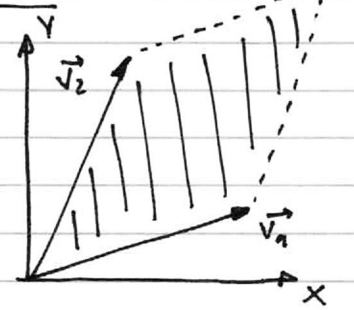
$$\det \begin{bmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & & \vdots \\ a_{N1} & \cdots & a_{NN} \end{bmatrix} = \sum a_{1i} (-1)^{i+1} \det (\underbrace{A_{i1}}_{\text{Matrix resulting of eliminating column } i \text{ and row } 1.})$$

example:

$$\det \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = a_{11} \det \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} + \\ - a_{12} \det \begin{bmatrix} a_{21} & a_{32} \\ a_{31} & a_{33} \end{bmatrix} + \\ + a_{13} \det \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

→ Geometric Interpretation:

in 2D:



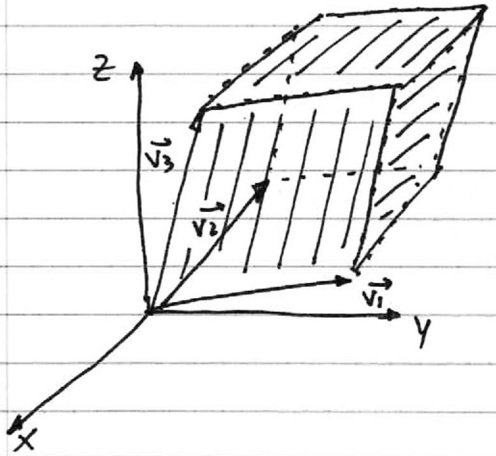
$$\det \left[\vec{v}_1 \ \vec{v}_2 \right] = \text{AREA parallelogram with vectors } \vec{v}_1 \text{ and } \vec{v}_2 \text{ as sides.}$$

NOTE: If $\det = 0 \Rightarrow$ Area $= 0$

$\Downarrow$

Both vectors are aligned.

in 3D:



$\det\left[\vec{v_1}\ \vec{v_2}\ \vec{v_3}\right]$ = VOLUM parallelepiped with $\vec{v_1}$, $\vec{v_2}$ and $\vec{v_3}$ as edges.

NOTE: This implies that if det = 0
Then volume is zero, which means that the three vectors are in a plane, so:
- They are linearly dependent.
- They do not span all 3D space.

→ Properties:

- $\det(A^T) = \det(A)$
- $\det(A^{-1}) = \dfrac{1}{\det(A)}$
- $\det(A \cdot B) = \det(A) \cdot \det(B)$
- $\det(R) = +1$ if R is a rotation matrix.
- $\det\left[\vec{v_1} \cdots K \cdot \vec{v_i} \cdots \vec{v_N}\right] = K \cdot \det\left[\vec{v_1} \cdots \vec{v_i} \cdots \vec{v_N}\right]$
- $\det(K \cdot A) = K^N \det(A)$ with N the dimension of A.