MIT - Mechanical Engineering Department

2.12 Introduction to Robotics - Fall 2016 - HW3

Released: September 27, 2016 Due: October 5, 2016

Notes:

- Turn in individual problems in **separate pages** and put your name on **each page**.
- Show the process to get to the answer and not just the solution.
- Clearly highlight solutions, i.e., draw a rectangle around them.

Problem 1

Differential Motion of an RRR non-planar Mechanism

The robot in Figure 1 is a non-planar RRR articulated mechanism with joint coordinates $\vec{q} = (\theta_1, \theta_2, \theta_3)$. We are interested in controlling the position $\vec{p} = (x_e, y_e, z_e)$ of its end-effector.

a. Obtain the Jacobian that relates $\dot{\vec{p}}$ and $\dot{\vec{q}}$ via direct differentiation of the direct kinematic equations.

b. Obtain the Jacobian for $(\theta_1 = \frac{\pi}{2}, \theta_2 = \frac{\pi}{6}, \theta_3 = \frac{-2\pi}{3})$ and its determinant. Assume $l_1 = l_2 = 1$.

c. Find the joint velocities that generate the end effector velocity $(v_x = 1, v_y = 2, v_z = 0)$ for the configuration in part b. Again assume $l_1 = l_2 = 1$.

d. Find all singular configurations of the manipulator.

e. Sketch the singular configurations from part d. and clearly indicate in which direction/directions the end-effector cannot move.

f. We know the endpoint can be placed anywhere in the workspace of the robot, but the above analysis says that it cannot be moved in certain directions, are these statements in conflict? Explain.

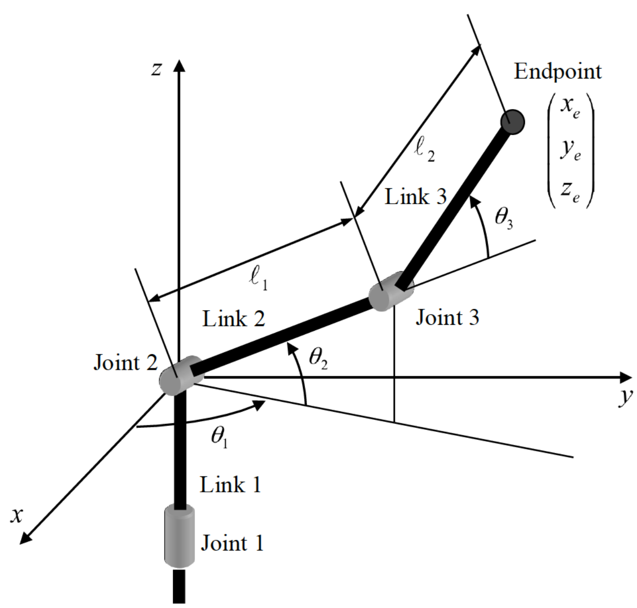

Figure 1: Schematic of a three dof articulated robot.

Problem 2

Slider-Crank Mechanism

[This problem is adapted from "A Mathematical Introduction to Robotic Manipulation" by Murray et. al.]

Consider the slider-crank mechanism in Figure 2 where a piston slides right-left inside a guide.

a. Calculate the number of degrees of freedom of the mechanism. You may use the *Gruebler Formula* for planar mechanisms given by:

$$DOF = 3 \cdot (n-1) - 2l - h \tag{1}$$

where n is the number of rigid bodies (including the ground), l is the number of lower pairs (joints that only allow for one dof motion) and h is the number of higher pairs (joints that allow 2 or more dof motion).

b. Show that we can relate the velocity of the slider to the angles (θ_1, θ_2) and the crank rotational velocity $\dot{\theta}_1$ as:

$$\dot{d} = -l_1 \sin\theta_1 \frac{l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2)}{l_2 \cos(\theta_1 + \theta_2)} \dot{\theta}_1 = J\dot{\theta}_1$$

c. Evaluate the singularities of the mechanism and draw the configurations at which this happens. Does the Jacobian always exist? If not explain and sketch a figure of this scenario.

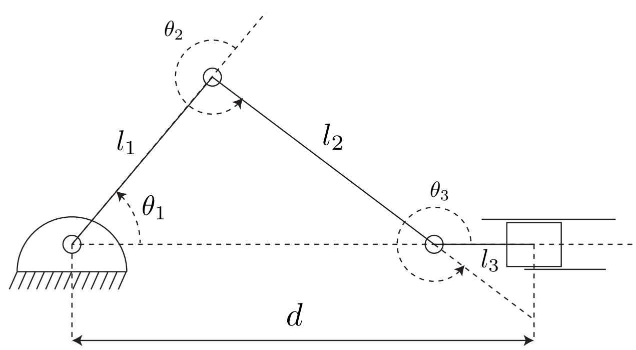

Figure 2: Slider Crank Mechanism.

Problem 3

Soccer Kick

Consider the one DOF leg robot in Figure 3 where the leg first pulls back and then kicks a ball during its forward motion. The leg and foot are all one rigid body, parametrized by the angle θ measured from the vertical line. At time $t = 0$s the leg is vertical ($\theta = 0$) with the tip of the foot at the origin of the reference frame $\{O, (x, y)\}$. At $t = 1$s a ball arrives at the origin O, where the leg is supposed to kick it.

a. We use a cubic spline function $\theta(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$ to describe a continuous trajectory in joint space. Find the parameters of the spline (a_i) so that the these initial and final conditions are satisfied:

- Initial Conditions: $\theta(0) = 0\,\text{rad},\;\; \dot{\theta}(0) = 0\,\text{rad/sec}$
- Final Conditions: $\theta(1) = 0\,\text{rad},\;\; \dot{\theta}(1) = 3\,\text{rad/sec}$

b. Plot the spline from part a and find how far the leg is pushed backward before the ball is kicked.

c. Now we want to extend the trajectory obtained in part a. After kicking the ball, the leg continues to move forward and at time $t = 2$s arrives at a destination with zero velocity. Find the two segments of the cubic spline connected together that satisfy the following conditions:

- Initial Conditions: $\theta(0) = 0\,\text{rad},\;\; \dot{\theta}(0) = 0\,\text{rad/sec}$
- Waypoint Conditions: $\theta(1) = 0\,\text{rad},\;\; \dot{\theta}(1) = 3\,\text{rad/sec}$
- Final Conditions: $\theta(2) = 1\,\text{rad},\;\; \dot{\theta}(2) = 0\,\text{rad/sec}$

d. Let's consider a different way of connecting the two spline segments at the waypoint. Instead of specifying the velocity at the waypoint, we would like both the velocity and the acceleration of the trajectory to be continuous at the waypoint so that there are no sharp transitions. Obtain the parameters (a_i) for the 2 spline segments that satisfy these conditions.

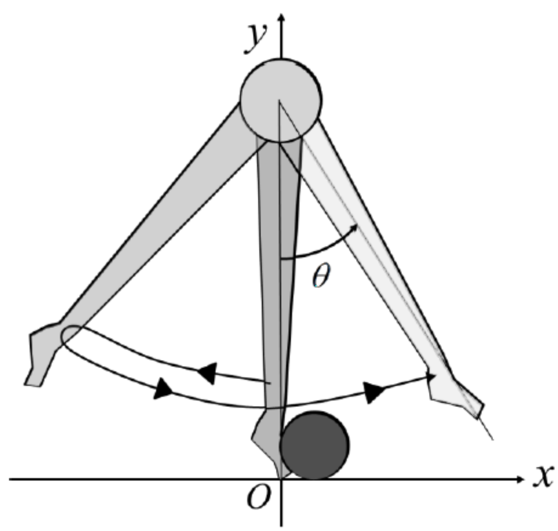

Figure 3: 1 DOF robot kicking the ball.

Problem 4

Optimal Inverse Differential Kinematics for Redundant Manipulators [2.120 Only]

We can control cartesian velocities of a manipulator by solving the Jacobian equation:

$$\text{Given } \dot{p} \quad \rightarrow \quad \text{find } \dot{q} \quad \text{s.t. } \dot{p} = J \cdot \dot{q}$$

In simple cases, we can solve the equation by inverting the Jacobian matrix $\dot{q} = J^{-1} \cdot \dot{p}$.

We want to look now at cases when it is not possible to invert the Jacobian, in particular, for redundant manipulators. Since these provide multiple solutions to the Jacobian equation, we can impose extra conditions to resolve the indeterminacy. A common strategy is to look for a solution that minimizes the effort of the actuators, modeled as a quadratic cost function on their velocities $G(\dot{q}) = \dot{q}^T W \dot{q}$, where W is a positive definite weighting matrix. More precisely, we reformulate the problem as:

$$\text{Given } \dot{p} \quad \rightarrow \quad \text{find } \dot{q} \quad \text{s.t. } \min_{\dot{q}} \ \dot{q}^T W \dot{q} \ \text{ and } \ \dot{p} = J\dot{q}$$

a. Use a Lagrange multiplier to show that the solution is given by $\dot{q} = W^{-1} J^T (J W^{-1} J^T)^{-1} \dot{p}$.

b. Consider now the manipulator in Figure 4 with Jacobian:

$$J = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 1 & 1 & 1 \end{bmatrix}$$

where $c_1 = \cos\theta_1, c_{12} = \cos(\theta_1 + \theta_2)$ and so on. We want its endpoint to move from point A to point B along the Y axis at a constant speed of 10 cm/s. Assuming that link 3 is kept parallel to the X axis, compute the angular velocities of the three joints and plot them as a function of time.

c. Does the manipulator Jacobian become singular during the motion? If so determine the singular configuration and the direction along which the arm cannot move.

d. If no condition is imposed on the orientation of link 3 during the motion starting at A, determine the joint velocities at point A that minimize the squared norm $v^2 = \dot{\theta}_1^2 + \dot{\theta}_2^2 + \dot{\theta}_3^2$.

e. Now compute the entire trajectories of each joint and plot their velocities while going from A to B but freeing the orientation of link 3 and minimizing the norm $v^2 = \dot{\theta}_1^2 + \dot{\theta}_2^2 + \dot{\theta}_3^2$.

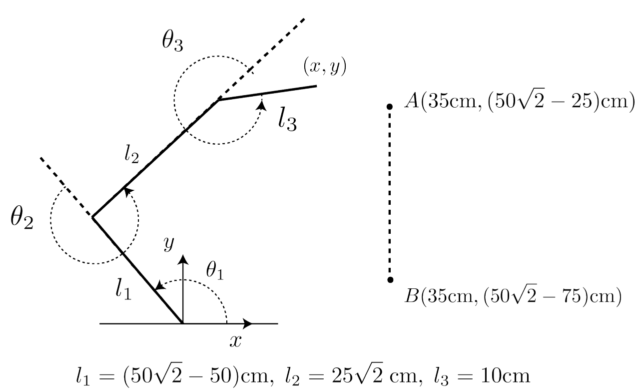

Figure 4: 3 Link manipulator going through a possible singularity.