HW 4:

P1: Code is provided

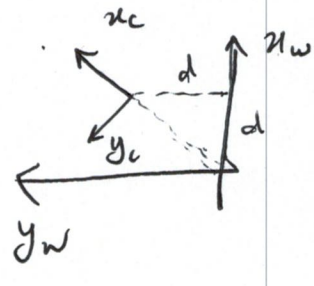
P2:

a) It is simple to see:

$$ {}^{w}_{c}T = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 & d \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & d \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

b) Full camera matrix is given by:

$$ C_M = \begin{bmatrix} s_x & 0 & u_{cam} \\ 0 & s_y & v_{cam} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underbrace{\left[\begin{array}{ccc|c} & R_z(-\theta) & & \begin{matrix} -d\sqrt{2} \\ 0 \\ 0 \end{matrix} \\ \hline 0 & 0 & 0 & 1 \end{array}\right]}_{{}^{c}_{w}T} $$



Now using numbers in the problem set

$s_x = s_y = 1000$ pixels/cm

$u_{cam} = 2000$ pixels

$v_{cam} = 1500$ pixels

$\theta = 45$

$d = 5$ m

$f = 2$ or 2×10^{-3}

Credit is awarded for any choice of units as long as they are consistent

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P1: Code is provided

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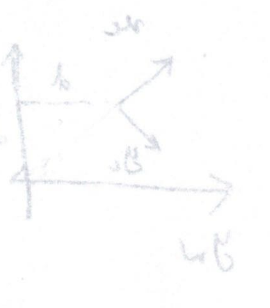
a) It is simple to see:

$$^{w}_{c}T = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & d \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & d \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) Full camera matrix is given by:

$$C_m = \begin{bmatrix} S_x & 0 & u_{cam} \\ 0 & S_y & v_{cam} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \underbrace{\begin{bmatrix} R_z(-\theta) & \begin{matrix} -d\sqrt{2} \\ 0 \\ 0 \end{matrix} \\ 0 \;\; 0 \;\; 0 & 1 \end{bmatrix}}_{^{c}_{w}T}$$



Now using numbers in the problem set

$S_x = S_y = 1000 \ \frac{\text{pixels}}{\text{cm}}$

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$f = 2$ or 2×10^{-2}

Credit is awarded for any choice of units as long as they are consistent

c) To find the pose of the points in world frame we use the camera matrix (You can use similarity triangles if you like):

$$\begin{Bmatrix} u \\ v \\ s \end{Bmatrix} = \begin{bmatrix} \frac{s_x f}{\sqrt{2}} & \frac{s_x f}{\sqrt{2}} & u_c & -d\sqrt{2}\, sf \\ \frac{-s_x f}{\sqrt{2}} & \frac{s_x f}{\sqrt{2}} & v_c & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \\ w \end{Bmatrix}$$

here we exploited $s_x = s_y$

$$\Rightarrow \quad u = \frac{s_x f}{\sqrt{2}}(x + y) + u_c z - d\sqrt{2}\, w s f$$

$$v = \frac{s_x f}{\sqrt{2}}(-x + y) + v_c z$$

$$s = w$$

Set $w = 1$ & $z = 10$

Then we have two unknowns (x, y) and 2 equations:

If $\begin{cases} s_x = 10^5 \\ f = 2 \times 10^{-3} \\ z = 10 \\ u = 1000 \\ v = 1000 \end{cases} \Rightarrow \begin{cases} x = -12.7 \\ y = -111.7 \\ z = 10 \end{cases}$

For ${}^{w}P_2 = \begin{bmatrix} -5.6 \\ -104.6 \\ 10 \end{bmatrix}$

For ${}^{w}P_3 = \begin{Bmatrix} -12.7 \\ -104.6 \\ 10 \end{Bmatrix}$

c) To find the pose of the points in world frame we use the Camera matrix (You can use Simulink triangles if you like):

$$\begin{Bmatrix} u \\ v \\ 1 \end{Bmatrix} = \begin{bmatrix} \frac{S_x f}{z_c} & \frac{S_x f}{z_c} & u_c & -d_x z_c \\ -\frac{S_y f}{z_c} & \frac{S_y f}{z_c} & v_c & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{Bmatrix} x \\ y \\ z \\ \omega \end{Bmatrix}$$

Here we exploited $S_x = S_y$

$$u = \frac{S_x f}{z_c}(x - y) + u_c z - d_x z_c \omega f$$

$$v = \frac{S_x f}{z_c}(-x + y) + v_c z$$

$$f = \omega$$

Set $\omega = 1$ & $z = 1.0$

Then we have two unknowns (x,y) and 2 equations:

If $\begin{cases} S_x = 1.5 \\ f = 2 \times 10^{-3} \\ z = 1.0 \\ u = 1000 \\ v = 1000 \end{cases}$ $\Rightarrow$ $x = -12.7$, $y = -11.7$

For u_3: $\begin{Bmatrix} -5.6 \\ -10.6 \\ 1.0 \end{Bmatrix}$

For u_3: $\begin{Bmatrix} -12.7 \\ -11.6 \\ 1.0 \end{Bmatrix}$