MIT - Mechanical Engineering Department

2.12 Introduction to Robotics - Fall 2016 - HW4

Released: October 5, 2016 Due: October 12, 2016

Notes:

- Turn in individual problems in **separate pages** and put your name on **each page**.
- Show the process to get to the answer and not just the solution.
- Clearly highlight solutions, i.e., draw a rectangle around them.

Problem 1

Potential Fields for Path Planning

In this problem we will implement a simple planar path planning algorithm to move a robot from configuration A to configuration B while avoiding obstacles, and without manually specifying waypoints. We will assume that either the robot is a point that can move independently in x and y, or more generally, that the planning problem is formulated in the configuration space of a robot with 2 degrees of freedom. We will focus on an approach that uses potential fields to attract the robot to its goal configuration while it is being repelled by obstacles.

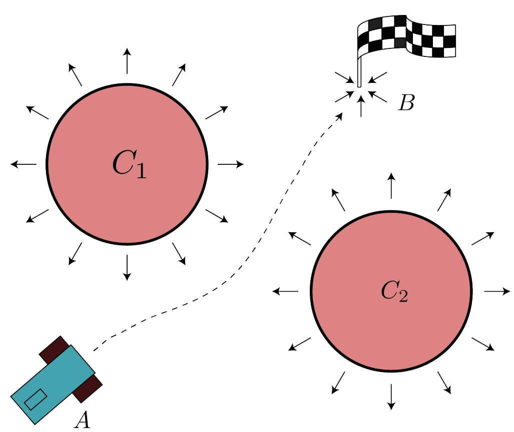

Figure 1: Attractor (Goal, point B) and Repellers (Obstacles, C_1 and C_2).

To represent attractors and repellers we use potential functions, i.e. scalar functions that naturally have a maximum or minimum that can be used to pull or repel. In this problem we use a Gaussian function. At the goal we place a negative Gaussian to create a bowl and at the obstacles we place positive Gaussians to create mountains. The total potential field is the sum of all potentials. The expressions for a negative Gaussian and its gradient are:

$$f(x) = -\frac{1}{\sqrt{(2\pi)^n|\Sigma|}}\exp\left(-\frac{1}{2}(x-u)^T\Sigma^{-1}(x-u)\right)$$

$$\frac{\partial f(x)}{\partial x} = -f(x)\Sigma^{-1}(x-u)$$

where $u_{n\times 1}$ is the center of the Gaussian, $\Sigma_{n\times n}$ is the covariance matrix of the Gaussian (a measure of how wide the Gaussian is in each dimension), $|\Sigma|$ is the determinant of the covariance matrix, and n is the dimension of the space.

To plan a path through a potential field we follow the gradient of the field, with the idea being that the gradient of a repeller potential pushes away from its center, and the gradient of an attractor potential pull to its center. If we construct the potential field intelligently, the robot can navigate to the goal simply by following the gradient of the total potential function. The gradient descent algorithm to find the minimum of a function $f(\cdot)$ begins from a guess solution x_0 and iterates by stepping like:

$$x_{k+1} = x_k - \alpha \frac{\partial f}{\partial x}(x_k), \quad k = 0, 1, 2, ...$$

where α is a hand picked step size and it terminates when it stops making significant progress on decreasing the value of f.

a. Consider the 1-dimensional negative Gaussian:

$$f(x) = -\frac{1}{\sqrt{(2\pi)^n |\Sigma|}} \exp\left(-\frac{1}{2}(x-u)^T \Sigma^{-1} (x-u)\right)$$

and set $u = 2$ and $\Sigma = 1$. Plot this function and, from the initial guess $x_0 = 0$ apply the gradient descent algorithm. Show the progression of the algorithm converging to the minimum of the function.

b. Consider the scene in Fig. 1, where $A = (x, y) = (-0.5, -0.5)$ is the start, $B = (x, y) = (0.75, 0.75)$ is the goal, and C_1 and C_2 are the obstacles we want to navigate around. In this problem the obstacles are modeled as circles in the plane of the motion of the robot with radii 0.5 and the centers respectively at $(-0.1, 0.23)$ and $(1.5, -0.5)$. Construct the total potential field and plot it with MATLAB's `contour` with:

$$u_B = \begin{bmatrix} 0.75 \\ 0.75 \end{bmatrix}$$
$$\Sigma_B = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}$$

and:

$$u_{C_1} = \begin{bmatrix} -0.1 \\ 0.23 \end{bmatrix}$$
$$u_{C_2} = \begin{bmatrix} 1.5 \\ -0.5 \end{bmatrix}$$
$$\Sigma_{C_1} = \Sigma_{C_2} = \begin{bmatrix} 0.09 & 0 \\ 0 & 0.09 \end{bmatrix}$$

c. Start with the robot at A, apply the gradient descent algorithm, and verify that the robot will take a path that reaches B and avoids the obstacles.

d. Change the covariance Σ of the potential functions. Can you generate a case in which the robot does not reach the goal?

Problem 2

From Pixels to Points

We are interested in registering points seen by a camera to a known global reference frame. Consider Fig. 2, where $\{O_w - (x_w, y_w, z_w)\}$ is the world reference frame and $\{O_c - (x_c, y_c, z_c)\}$ is the camera reference frame. The camera sees three points on its image plane with coordinates (in pixels):

$$\begin{aligned} p_1 &= (u_c = 1000, v_c = 1000), \\ p_2 &= (u_c = 3000, v_c = 1000), \\ p_3 &= (u_c = 2000, v_c = 2000) \end{aligned}$$

Assume that $d = 5$m, $\theta = 45^o$ and the camera is an ideal pinhole with focal length $f = 2$. Note that (x_c, y_c) are in the same plane as (x_w, y_w). The camera CCD has 1000 pixels per 1 centimeter and dimensions 4 by 3 centimeters along u and v and the camera origin is at the center of the CCD.

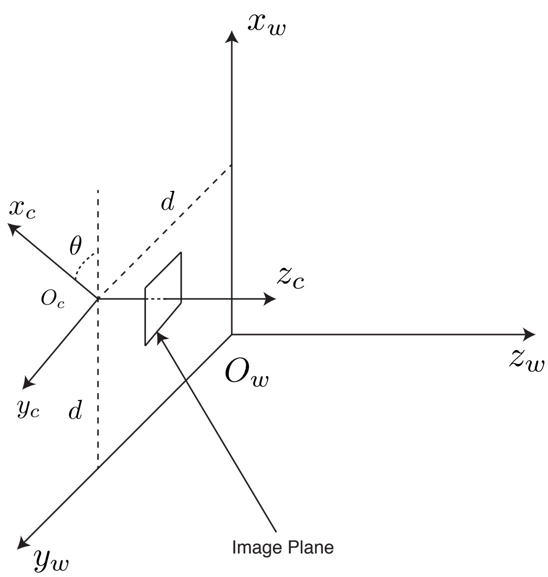

Figure 2: Camera and world framers.

a. Find the homogeneous transformation from the camera frame to the world frame ${}^w_cT$.

b. Derive the complete camera matrix and evaluate it with the given numbers.

c. Assume that the points are in reality 10 meters away along the z_c direction, find the position of each point with respect to the global coordinate frame $\{O_w - (x_w, y_w, z_w)\}$.

Problem 3

Potential Fields for Path Planning [2.120]

In problem 1 we considered circular obstacles, and used negative Gaussian functions to represent them. For more complex obstacles, specially in their representation in configuration space, the choice of a Gaussian may not be ideal.

a. Propose a potential function representation for simple objects such as squares and triangles, evaluate their gradient, and providing a rationale as to why it is a good choice.

b. In theory, any strictly convex and differentiable R^n to R function can be used as a potential function in constructing the potential field, one such example is the quadratic form $x^T \cdot Q \cdot x$, why would this choice not always yield desirable results. [Hint: Plot a 1-dimentional negative Gaussian and a quadratic on top of each other and see what happens for different initial conditions.]