Problem 1

Generating a Desired Force in a Static Manipulator

A planar robot with three revolute joints is shown below. Let $\theta_i$ and $l_i$ be the angle of joint $i$ and the length of link $i$, respectively, and $(x_e, y_e, \phi_e)$ be the end-effector position and orientation viewed from the base coordinate frame, as shown in the figure. Linear forces $F_x$ and $F_y$ and moment $N_z$ act on the end-effector. Answer the following questions:

1. At arm configuration of $(\theta_1 = 135^\circ, \theta_2 = 45^\circ, \theta_3 = 225^\circ)$ obtain the $3 \times 3$ Jacobian matrix relating the end-effector position and orientation to joint displacements.

2. We want to generate an endpoint force of $(F_x = 10\text{N}, F_y = -2\text{N}, N_z = 0.2\text{Nm})$. Obtain the equivalent joint torques needed for generating the endpoint force. Ignore friction and gravity.
Problem 2

Principle of Virtual Work - Bat Robot

Consider the cool (and potentially useful for Halloween) bat robot below. This robot has biologically inspired with elastic joints that are activated with artificial muscle actuators. Figure 2 shows the cross-sectional view of the wing structure. Each wing consists of two rigid bodies, Link 1 (line \(AB\)) and Link 2 (line \(BE\)), connected at Joint 2 (Point \(B\)). A coil spring of spring constant \(k\) is inserted between Points \(C\) and \(D\), generating a contracting force \(F_s = k(h - h_0)\), as shown in the figure. Parameter \(h_0\) is the unstrung length of the spring, and \(h\) is the distance between \(C\) and \(D\), both of which are at distance \(b\) from Joint 2. An artificial muscle actuator is attached to one end of Link 1 (Point \(A\)), which rotates around Joint 1. Using the joint angles \(\theta_1\) and \(\theta_2\), link lengths \(l_0\), \(l_1\) and \(l_2\) shown in the figure, answer the following questions. Ignore friction and use the Energy Method.

![Bat Robot Schematic](image)

1. What are a complete and independent set of generalized coordinates that locate the single wing system described above?

2. Obtain the length of the coil spring \(h\) as a function of joint angle \(\theta_2\). Also obtain the differential relationship between virtual displacements \(\delta h\) and \(\delta \theta_2\).

3. When the bat robot glides, aerodynamic lift acts along the wing. For the sake of simplicity the lift is approximated to two forces \(F_b\) and \(F_e\) acting at Points \(B\) and \(E\), respectively. Using the Principle of Virtual Work, obtain the linear actuator force \(f\) needed for bearing the forces \(F_b\) and \(F_e\). Assume no friction and no gravity.

4. Obtain the joint angle \(\theta_2\) when the system with given forces \(F_b\), \(F_e\), and the actuator force \(f\) is in equilibrium. For simplicity set the parameter \(h_0 = 0\) to zero and assume \(\theta_1 = 0\).
Problem 3

Hybrid position/force Control Constraints

Shown below is an office robot drying ink with blotting paper attached to a semicircular roller of radius $R$. The roller should not slide but roll on the paper in order to avoid smearing the wet signature. Assuming that the process is quasi-static and frictionless, we want to perform the task using hybrid position/force control. Obtain natural and artificial constraints in terms of velocities and forces at the robot endpoint $E$. Describe the constraints with respect to the coordinate system $\{O - xyz\}$ that is fixed to space but is at Point $E$ at the instant shown. Note that Point $E$ is in the middle of the top surface of the semicircular roller and that the $x$ and $y$ axes are parallel to the sheet of paper. Is the rolling-contact requirement a natural constraint or an artificial constraint? [The key is to differentiate constraints that physics dictates, i.e. natural constraints, from the type of trajectories that you want the robot to follow in order to accomplish a given task, i.e. artificial constraints.]

Figure 3: Office robot of President Rafael Reif drying ink with blotting paper.