Massachusetts Institute of Technology
Department of Mechanical Engineering

2.12/2.120 Introduction to Robotics
Solutions of the Mid-Term Examination
October 22, 2014

Closed-Book. Two sheets of notes are allowed.
Show how you arrived at your answer.
Do not leave multiple answers. Indicate which one is your correct answer.

Problem 1 (50 points + 5 point extra)

A goalie must be able to react to a flying ball quickly and generate a high acceleration of the body. A 2.12 project team decided to use a parallel manipulator, where the actuators are not placed at the arm links, but are fixed to the base, so that the mass of the actuator may not be a load for other actuators. Figure 1 shows the schematic of a 2 DOF arm, where two actuators are fixed to the base link. Actuator 1 is directly connected to Link1, while the torque of Actuator 2 is transmitted through the belt-pulley mechanism to Link 2. Answer the following questions.

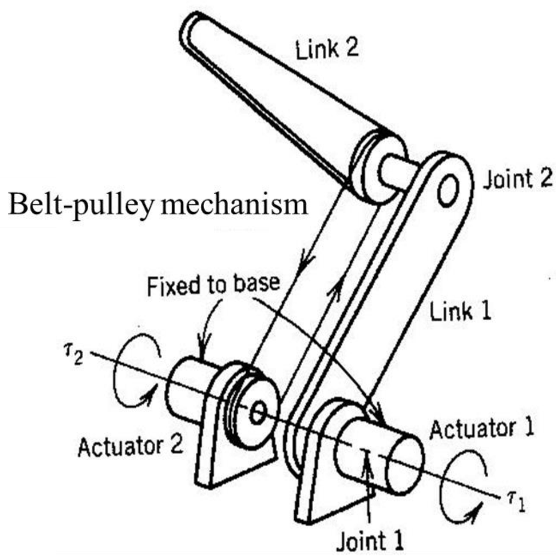


Figure 1 Parallel drive mechanism

a). The project team first looked at the gearing of each joint, and obtained an optimal gear ratio to maximize the angular acceleration. As shown in Figure 2, the inertia of the motor rotor including the motor shaft and the small gear is $I_m = 2.5\times10^{-3}$ kg $\cdot$ m^2, whereas that of the arm link including the large gear is $I_l = 1.0$ kg $\cdot m^2$. Obtain the gear ratio $1:r, (r>1)$ that maximizes the angular acceleration of the arm link, when a maximum armature voltage is applied to the motor while its angular velocity is small.

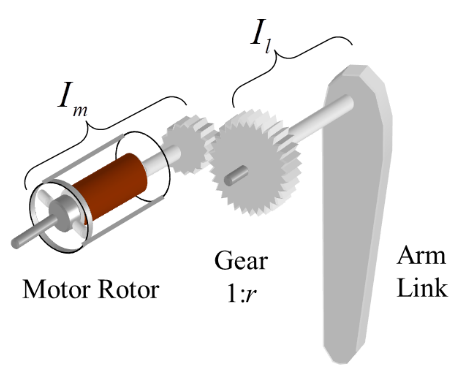


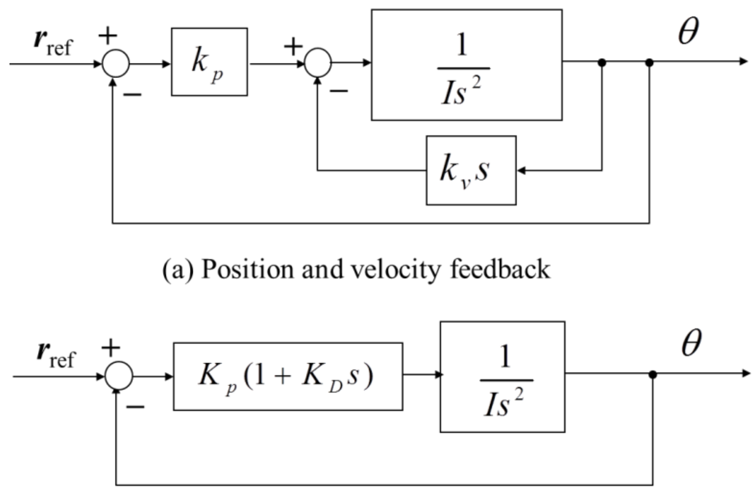
Figure 2 Single-axis joint

b). Two members of the project team proposed different control laws for the single joint drive system. Figure 3-(a) shows a position and velocity feedback law, while Figure 3-(b) is a Proportional and Derivative control. Using proper values for k_D, K_V, draw a root locus for each control system.

b*). [Extra Credit] Discuss which control law is better for improving rise time while keeping overshoot small.

c). Figure 4 shows two different sets of generalized coordinates proposed by two members of the team. In (a), variable θ_2 is the relative angle of Link 2 to Link1, while in (b) variable ϕ_2 is the angle of Link 2 measured from the *x* axis. Both sets of generalized coordinates can locate the system completely, and are independent. Notice that each actuator displacement is measured by an encoder attached to the motor shaft. Which set of generalized coordinates can represent the encoder readings correctly? Using the generalized coordinates representing the encoder readings, obtain endpoint coordinates x_e, y_e.

d). Obtain motor torques, τ_{m1}, τ_{m2}, for generating endpoint forces, F_x, F_y, at rest at a given arm configuration. Assume no friction and no gravity. Also assume that both motors have the same gear ratio, 1: *r*.



(b) Proportional and Derivative control

Figure 3 Two control systems proposed

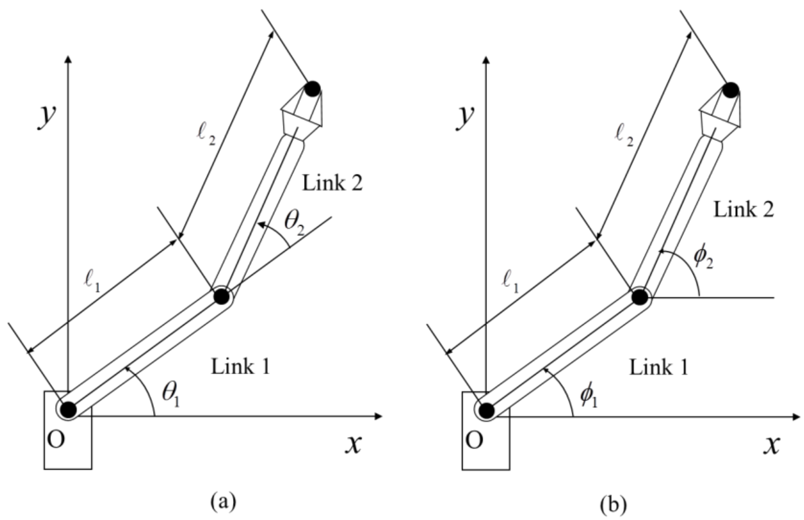


Figure 4 Two sets of generalized coordinates

e). Obtain the total power consumption at both motors when exerting the endpoint forces, F_x, F_y. Both motors have the same torque constant and armature resistance, K_t, R, respectively.

Solution

1-a) Let *f* be the force acting at the gear teeth contacting each other. From the free-body diagram in Figure S-1,

$$I_m \dot{\omega}_m = \tau_m - 1 \cdot f, \quad I_\ell \dot{\omega}_\ell = r \cdot f \qquad (1\text{-}1)$$

Eliminating the constraint force *f*,

$$(I_\ell + r^2 I_m)\dot{\omega}_\ell = r\tau_m$$

$$\dot{\omega}_\ell = g(r)\tau_m, \quad g(r) = \frac{r}{I_\ell + r^2 I_m}$$

The necessary condition for the maximum of acceleration is obtained by

$$\frac{dg(r)}{dr} = \frac{1\cdot(I_\ell + r^2 I_m) - r\cdot 2I_m r}{(I_\ell + r^2 I_m)^2} = 0$$

Therefore, $r^2 I_m = I_\ell$. The maximum of acceleration is achieved when the load-side inertia is the same as the effective inertia of the motor-rotor side. This is referred to as "impedance matching".

$$\therefore r = \sqrt{\frac{I_\ell}{I_m}} = \sqrt{\frac{1}{2.5\times 10^{-3}}} = 20$$

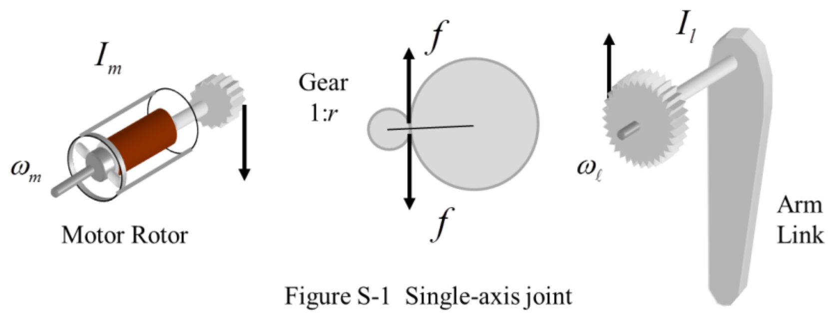


Figure S-1 Single-axis joint

1-b). The position and velocity feedback system can be reduced to the block diagram shown in Figure S-2 (a). The loop transfer function is $L(s) = \dfrac{1}{s(Is + k_v)}$, and the open-loop poles are at

$p_1 = 0, \quad p_2 = -\dfrac{k_v}{I}$. The root locus is therefore given by Figure S-2 (b).

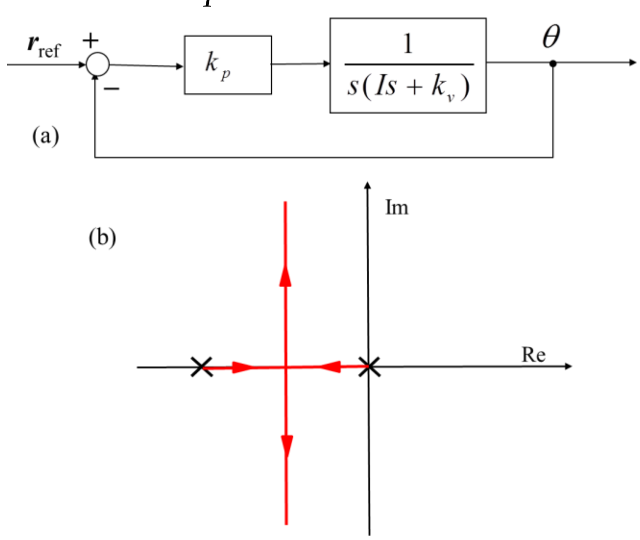


Figure S-2 Position and velocity feedback

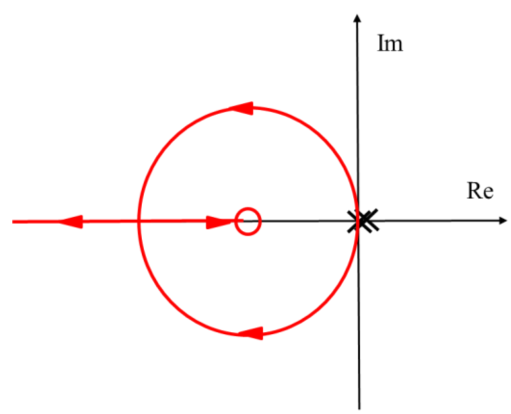


Figure S-3 Root locus of PD control system

The loop-transfer function of the PD control system is given by

$$L(s) = \frac{1 + K_D s}{Is^2}$$

The open-loop poles and zero are

$$p_{1,2} = 0, 0, \quad z = -\frac{1}{K_D}$$

Then, the root-locus is given by Figure S-3.

1-b*) Extra point

Both control systems contain both position and velocity feedback loops. The only difference is that the PD control system uses the derivative of reference input, while the position and velocity feedback has the derivative of the only joint displacement, i.e. the velocity. The input derivative increases the speed of response with a positive initial slope, $\frac{dq}{dt}|_{t=0} > 0$, while that of the position and velocity feedback is zero, $\frac{dq}{dt}|_{t=0} = 0$. See Figure S-4. Note that the step responses of the two systems have the same damping factor. The PD control tends to have a larger overshoot, but can be tuned with the derivative gain.

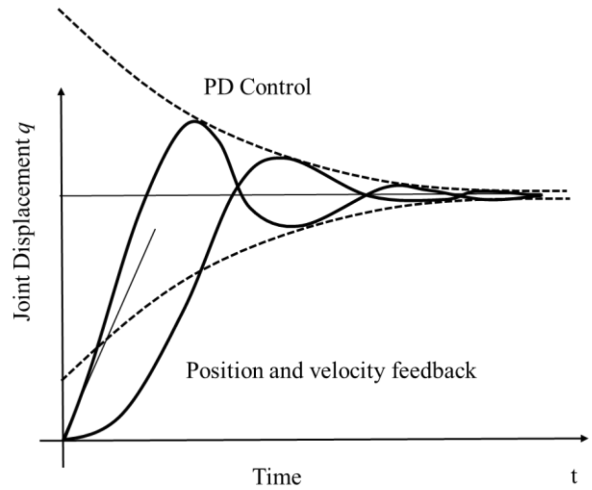


Figure S-4 Step responses

1-c). Since both motors are fixed to the base, the encoders measure angles relative to the base. Notice that θ_2 represents the angle relative to Link 1. Therefore, ϕ_1, ϕ_2 represent the correct encoder readings.

$$x_e = \ell_1 \cos\phi_1 + \ell_2 \cos\phi_2$$
$$y_e = \ell_1 \sin\phi_1 + \ell_2 \sin\phi_2$$

1-d). The differential kinematic relation is given by

$$\begin{pmatrix} \dot{x}_e \\ \dot{y}_e \end{pmatrix} = \begin{pmatrix} -\ell_1 s_1 & -\ell_2 s_2 \\ \ell_1 c_1 & \ell_2 c_2 \end{pmatrix} \begin{pmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{pmatrix}$$

Due to duality, the joint torques are given by

$$\begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} = \begin{pmatrix} -\ell_1 s_1 & \ell_1 c_1 \\ -\ell_2 s_2 & \ell_2 c_2 \end{pmatrix} \begin{pmatrix} F_x \\ F_y \end{pmatrix}$$

Therefore, the motor torques are:

$$\tau_{m1} = \frac{\tau_1}{r} = \frac{\ell_1}{r}(-F_x s_1 + F_y c_1)$$
$$\tau_{m2} = \frac{\tau_2}{r} = \frac{\ell_2}{r}(-F_x s_2 + F_y c_2)$$

1-e). Power consumption is $P = Ri^2 = \frac{R}{K_t^2}\tau^2$. The total power consumption is therefore given by

$$P_{total} = \frac{R}{K_t^2 r^2}\left[\ell_1^2(-F_x s_1 + F_y c_1)^2 + \ell_2^2(-F_x s_2 + F_y c_2)^2\right]$$

Problem 2 (20 points)

A self-driving electric vehicle is approaching a juice station. In order to properly park the vehicle, a trajectory must be designed such that the vehicle can arrive at the juice station at the right orientation. From the trajectory, angular velocity commands must be determined for both wheels, ω_R and ω_L. Let r be the radius of both tires and $2b$ the distance between the tires, as shown in Figure 5. The vehicle is initially at position A, which is the origin of the world

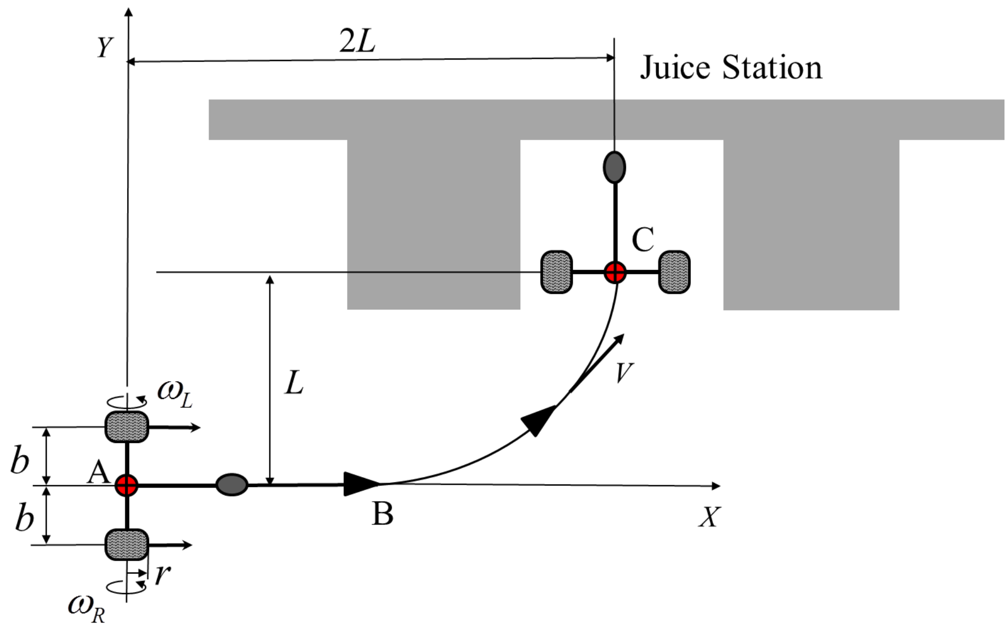


Figure 5 Self-driving car

coordinate frame *A-XY*. Initially the vehicle moves straight in the positive *X* direction for some distance and then changes the wheel velocities to follow the curved trajectory that leads the vehicle to the destination position C at $X = 2L$ and $Y = L$. At C the vehicle must face in the positive *Y* direction. The vehicle speed, i.e. the tangential velocity along the trajectory of the vehicle center point, must be kept constant at *V* for both straight and curved segments of the trajectory. Figure 6 shows the time profiles of ω_R and ω_L. Given *V* and *L* as well as *b* and *r*, find the following quantities.

a). Wheel angular velocities $\bar{\omega}_R = \bar{\omega}_L$ for the straight segment of the trajectory, and the ending time, t_1, of

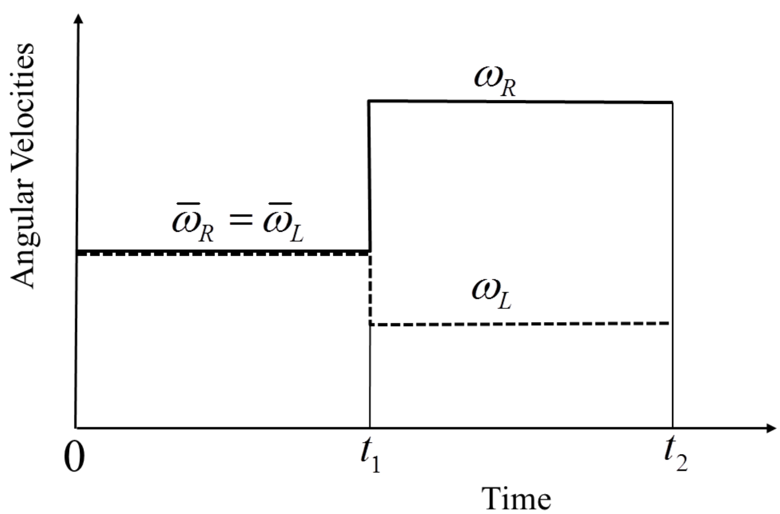


Figure 6 Wheel velocity commands

the straight segment. See Figure 6.

b). The individual wheel angular velocities, ω_R and ω_L, for the curved segment, and the ending time, t_2, of the curved segment.

<u>*Solution*</u>

2-a). Both ω_R and ω_L are constant during the second segment tracking. This implies that the trajectory is a <u>circle</u>. From the diagram the radius of the circular trajectory can be found *L*. Therefore, the distance from point A to point B is then *L*. Since the vehicle velocity is *V*,

$$t_1 = \frac{L}{V}$$

As the vehicle goes straight:

$$V = r\omega_R = r\omega_L, \quad \therefore \omega_R = \omega_L = \frac{V}{r} \; .$$

2-b). Since the radius of the curved trajectory is *L*,

$$\frac{\omega_L}{\omega_R} = \frac{L-b}{L+b}$$

The vehicle velocity is related to the wheel angular velocities as:

$$V = \frac{r}{2}(\omega_R + \omega_L) = \frac{r}{2}(1 + \frac{L-b}{L+b})\omega_R = \frac{rL}{L+b}\omega_R$$

$$\therefore \omega_R = \frac{L+b}{rL}V, \; \omega_L = \frac{L-b}{rL}V$$

The arc length of the curved trajectory is $L\frac{\pi}{2}$. The time required for tracking the curved trajectory at the given speed is $\Delta t = \frac{\pi L}{2V}$. Therefore,

$$t_2 = t_1 + \Delta t = \frac{L}{V}(1 + \frac{\pi}{2}) \; .$$

The following problem is for 2.120 students. For 2.12, it is an extra credit problem.

Problem 2-G (20 points)

The same vehicle as in Problem 2 is now parked in the opposite direction, as shown in Figure 7. The vehicle is initially at position A, which is at distance *L* away from the destination in both *X* and *Y* directions, and moves at a constant speed *V* along the first segment of the curved trajectory. The vehicle is then switched back at position B and follows the second segment of the curved trajectory backwards at the same constant speed *V*. The individual wheel angular velocities, ω_R and ω_L, are kept constant in each of the curved segments. Given parameters *L*, *V*, *b*, and *r*, find the following quantities.

a). Coordinates of switch back position B, X_B, Y_B.

b). Wheel angular velocities, ω_R and ω_L, for both curved segments.

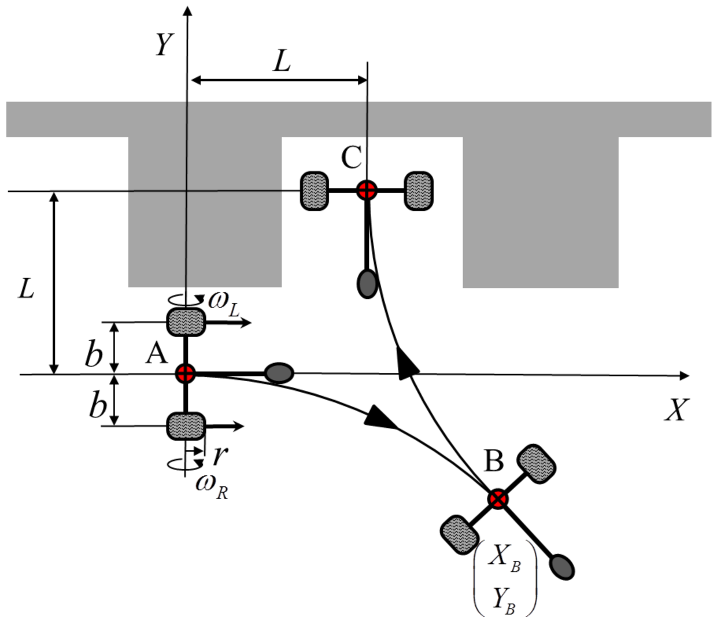


Figure 7 Switch back trajectory

<u>Solution</u>

2-G-a). Let points D and F be centers of rotation for the first and second segments of curved trajectories, respectively. As shown in Figure S-5, point D must be on the extension of the vehicle axle at its initial location, point A, and point F must be on the axil at point C. Note also that Points D and F must be on the extension of the axle at point B. These determine that points A, B, and C must be on the edges of the right triangle DEF, as shown in Figure S-5.

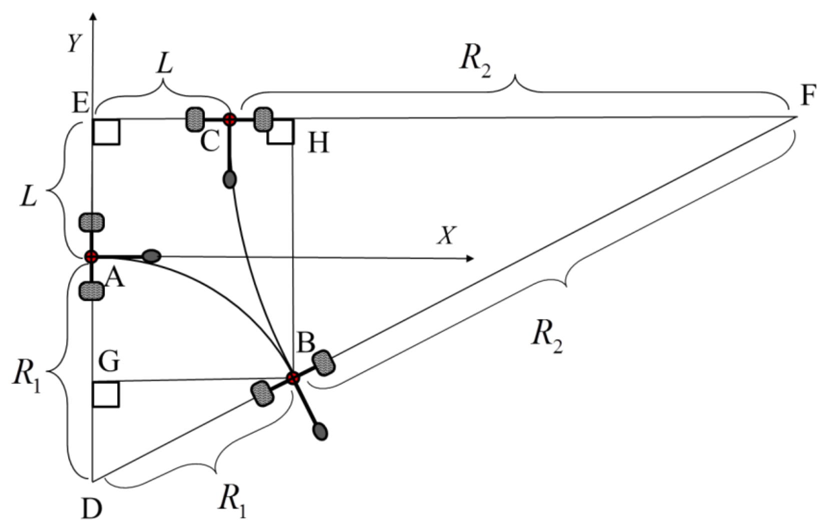


Figure S-5 Trajectory planning for Problem 2-G

From the Pythagorean Theorem we find that the following condition must hold for the radii R_1 and R_2 and parameter *L*,

$$(R_1+L)^2+(R_2+L)^2=(R_1+R_2)^2 \qquad (1)$$

This determines the relationship between radii R_1 and R_2. Solving (1) for R_2 yields:

$$R_2=\frac{R_1+L}{R_1-L}L \qquad (2)$$

Note that this holds for $R_1>L$. In the figure, lines BG and BH are perpendicular to edges DE and EF, respectively. Coordinate X_B, that is $\overline{EH}$, can be found by prorating $\overline{EF}=L+R_2$ into R_1 and R_2:

$$X_B=(L+R_2)\frac{R_1}{R_1+R_2}=\frac{2R_1^2}{R_1^2+L^2}L \qquad (3)$$

where (2) is used to obtain the final result. Similarly,

$$\overline{EG}=\overline{DE}\frac{R_2}{R_1+R_2} \qquad (4)$$

Therefore, coordinate Y_B is given by

$$Y_B=-(\overline{EG}-L)=-\frac{2LR_1}{R_1^2+L^2}L \qquad (5)$$

There are an infinite number of trajectories. If you pick a value of R_1 slightly larger than *L*, then R_2 becomes large, that is, the second segment is almost a straight line. If R_1 is much larger than *L*, then R_2 approaches *L*.

3-G-b). As before, the following two relationships allow us to determine the wheels angular velocities. Note, however, that the location of the center of rotation on the axle line is on the opposite direction. Therefore, we use - R_1 and - R_2. For the first segment,

$$\frac{\omega_L}{\omega_R}=\frac{-R_1-b}{-R_1+b},\quad V=\frac{r}{2}(\omega_R+\omega_L) \qquad (6)$$

Solving these yields

$$\omega_R=\frac{R_1-b}{R_1}\frac{V}{r},\quad \omega_L=\frac{R_1+b}{R_1}\frac{V}{r} \qquad (7)$$

Similar results are obtained for the second segment by replacing R_1 by R_2 in (7) and changing the sign to minus:

$$\omega_R=-\frac{R_2-b}{R_2}\frac{V}{r},\quad \omega_L=-\frac{R_2+b}{R_2}\frac{V}{r} \qquad (8)$$

Problem 3 (30 points for questions a, b, and c; + 20 points for questions d and e for 2.120 / extra credit for 2.12)

Figure 8 shows the kinematic structure of a 3 DOF striker robot designed by a term project team. It consists of three revolute joints connecting Links 0 through 3. Coordinate frame, $O - x_0 y_0 z_0$, is fixed to a vertical column to which the robot is secured. Coordinate frame $A - x_1 y_1 z_1$ is attached to Link 1 at point A, while frame $C - x_2 y_2 z_2$ is fixed to Link 2 at point C, and frame $E - x_3 y_3 z_3$ is attached to the end-effector, i.e. a soccer shoe. Joint 1 is rotation about the x_0 axis, while Joint 2 is about the y_1 axis and Joint 3 about the x_2 axis. All the joint angles are measured in a right hand sense. Angles OAB, ABC, BCD, and CDE are all the right angle, and the link dimensions are denoted as $\overline{OA} = \ell_0$, $\overline{AB} = \ell_1$, $\overline{BC} = \ell_2$, $\overline{CD} = \ell_3$, and $\overline{DE} = \ell_4$. Note that Figure 8 shows the posture of the striker robot at Home Position, where all the joint angles are zero: $\theta_1 = 0, \theta_2 = 0$, and $\theta_3 = 0$. Answer the following questions.

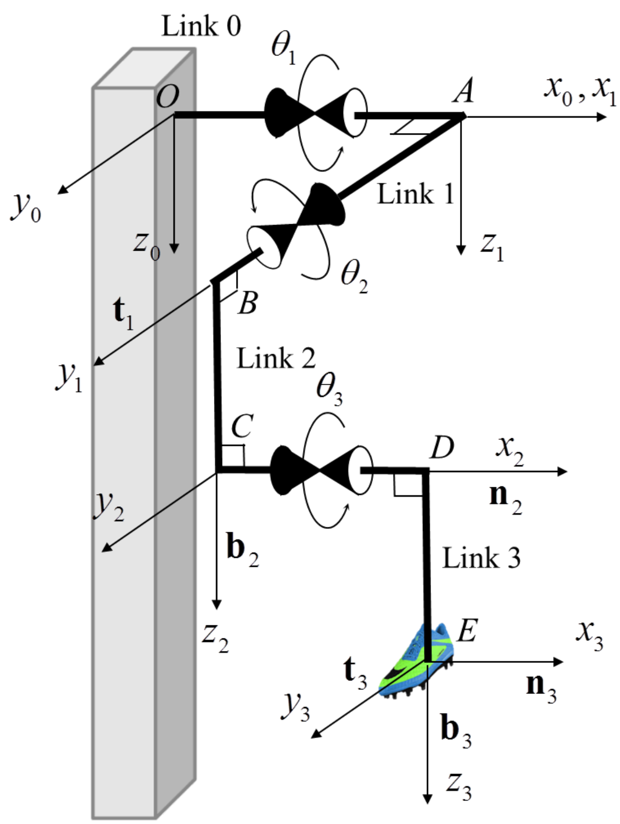


Figure 8 Striker robot design

a). Obtain the 3x3 rotation matrices associated with the individual joints of angles θ_1, θ_2, and θ_3, respectively.

b). Obtain unit vectors $\mathbf{t}_1, \mathbf{b}_2, \mathbf{n}_2$, and $\mathbf{b}_3$ viewed from the base frame that point in the directions of axes y_1, z_2, x_2, and z_3, respectively.

c). Obtain the endpoint coordinates using the unit vectors in part b).

The following questions are only for 2.120 students. For 2.12 they are for extra credit of max. 20 points.

d). Consider a robot structure where $\overline{AB} = \ell_1 = 0$, $\overline{CD} = \ell_3 = 0$, and $\overline{BC} = \overline{DE} = \ell$ with moveable ranges of joint angles, $-\pi \le \theta_1 < \pi, 0 \le \theta_2 \le \pi, -\pi \le \theta_3 < \pi$. The most important functional requirement for this striker robot is to kick a ball flying into a specific zone from the y_0 direction. Figure 9 shows the zone of possible ball locations within the frontal plane $O - x_0 z_0$. Can the robot generate an endpoint velocity in the positive y_0 direction within this zone of frontal plane? By sketching the Jacobian column vectors J_1, J_2, J_3, graphically explain whether it can

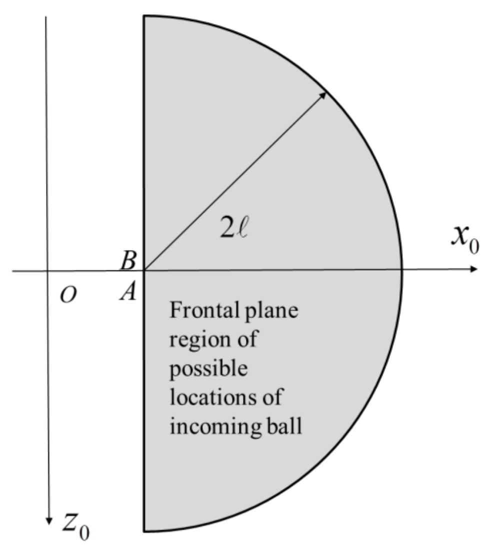


Figure 9 Frontal plane region of possible ball locations

kick the ball. Draw elevation and side views if necessary. Discuss limitations to this robot design.

d). Obtain the Jacobian matrix relating endpoint velocities $\dot{x}_e, \dot{y}_e, \dot{z}_e$ to joint velocities, $\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3$, for the robot structure in part c), and verify your answer to part c) quantitatively based on the properties of the Jacobian.

Solution

3-a). The first joint is rotation about the x_1 axis. Therefore,

$$R_x(\theta_1) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_1 & -s_1 \\ 0 & s_1 & c_1 \end{pmatrix} \tag{1}$$

Similarly for the second and the third joints,

$$R_y(\theta_2) = \begin{pmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{pmatrix}, \quad R_x(\theta_3) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & -s_3 \\ 0 & s_3 & c_3 \end{pmatrix} \tag{2}$$

3-b). The three unit vectors pointing in the directions of coordinate axes are column vectors in the associated rotation matrix. For the first frame,

$$\begin{bmatrix} \mathbf{n}_1^0 & \mathbf{t}_1^0 & \mathbf{b}_1^0 \end{bmatrix} = R_x(\theta_1), \quad \mathbf{t}_1^0 = \begin{pmatrix} 0 \\ c_1 \\ s_1 \end{pmatrix} \tag{3}$$

For the second frame viewed from the base frame,

$$\begin{bmatrix} \mathbf{n}_2^0 & \mathbf{t}_2^0 & \mathbf{b}_2^0 \end{bmatrix} = R_x(\theta_1)R_y(\theta_2) = \begin{pmatrix} c_2 & 0 & s_2 \\ s_1 s_2 & c_1 & -s_1 c_2 \\ -c_1 s_2 & s_1 & c_1 c_2 \end{pmatrix}, \quad \therefore \mathbf{n}_2^0 = \begin{pmatrix} c_2 \\ s_1 s_2 \\ -c_1 s_2 \end{pmatrix}, \quad \mathbf{b}_2^0 = \begin{pmatrix} s_2 \\ -s_1 c_2 \\ c_1 c_2 \end{pmatrix} \tag{4}$$

For the third frame,

$$\begin{bmatrix} \mathbf{n}_3^0 & \mathbf{t}_3^0 & \mathbf{b}_3^0 \end{bmatrix} = R_x(\theta_1)R_y(\theta_2)R_x(\theta_3) = \begin{pmatrix} * & * & s_2 c_3 \\ * & * & -c_1 s_3 - s_1 c_2 c_3 \\ * & * & -s_1 s_3 + c_1 c_2 c_3 \end{pmatrix}, \quad \therefore \mathbf{b}_3^0 = \begin{pmatrix} s_2 c_3 \\ -c_1 s_3 - s_1 c_2 c_3 \\ -s_1 s_3 + c_1 c_2 c_3 \end{pmatrix} \tag{5}$$

3-c). The endpoint can be reached by moving from the origin through points A, B, C, D, and E. The directions of the individual moves have been obtained above as unit vectors $\mathbf{n}_0^0, \mathbf{t}_1^0, \mathbf{b}_2^0, \mathbf{n}_2^0, \mathbf{b}_3^0$. Therefore,

$$\begin{pmatrix} x_e \\ y_3 \\ z_e \end{pmatrix} = \ell_0 \mathbf{n}_0^0 + \ell_1 \mathbf{t}_1^0 + \ell_2 \mathbf{b}_2^0 + \ell_3 \mathbf{n}_2^0 + \ell_4 \mathbf{b}_3^0 \tag{6}$$

or

$$\begin{aligned} x_e &= \ell_0 + \ell_2 s_2 + \ell_3 c_2 + \ell_4 s_2 c_3 \\ y_e &= \ell_1 c_1 - \ell_2 s_1 c_2 + \ell_3 s_1 s_2 - \ell_4 c_1 s_3 - \ell_4 s_1 c_2 c_3 \\ z_e &= \ell_1 s_1 + \ell_2 c_1 c_2 - \ell_3 c_1 s_2 - \ell_4 s_1 s_3 + \ell_4 c_1 c_2 c_3 \end{aligned} \tag{7}$$

3-d). For this design, the endpoint coordinates are reduced to:

$$\begin{aligned} x_e &= \ell_0 + \ell s_2(1+c_3) \\ y_e &= -\ell s_1 c_2(1+c_3) - \ell c_1 s_3 \\ z_e &= \ell c_1 c_2(1+c_3) - \ell s_1 s_3 \end{aligned} \quad (8)$$

We have to examine two points:

- Does the work space of the robot cover the region of incoming balls? In other words, can the robot reach all the locations in Figure 9 within the frontal plane?
- Can the robot generate an endpoint velocity in the positive y_0 direction?

For the first question, we can examine the existence of the inverse kinematics problem of (8). If the solution does not exist, that point is not reachable. Within the frontal region, the solution exists for most of the region. However, it does not exist for a segment of line along the x_0 axis. Let us examine this by setting $y_0 = 0, z_0 = 0$ in (8). This yields:

$$\begin{aligned} 0 &= -\ell s_1 c_2(1+c_3) - \ell c_1 s_3 \\ 0 &= \ell c_1 c_2(1+c_3) - \ell s_1 s_3 \end{aligned} \quad (9)$$

If $c_1 c_2(1+c_3) \neq 0$ and $s_1 s_3 \neq 0$, then dividing the first equation of (9) by the second equation,

$$\frac{s_1 c_2(1+c_3)}{c_1 c_2(1+c_3)} = \frac{c_1 s_3}{-s_1 s_3}, \quad \therefore -s_1^2 = c_1^2 \quad (10)$$

This is a clear contradiction, since $s_1^2 + c_1^2 = 1$. Therefore, the position on the x_0 axis is not reachable except for $\theta_2 = \pi/2,\ \theta_3 = 0, \pi$, points A and F in Figure S-6.

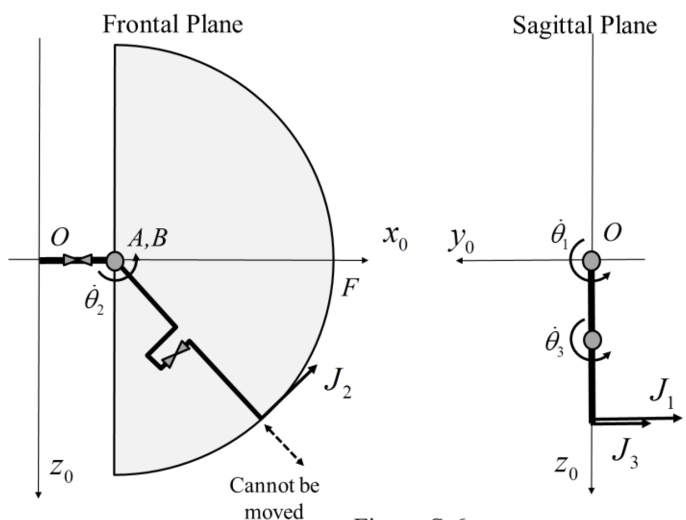


Figure S-6

For the second question we have to examine the Jacobian. If the robot configuration is non-singular, the endpoint can be moved in any direction at a non-zero velocity, which includes the positive y_0 direction. Therefore, we can examine only singular configurations within the region of incoming balls in the work space. Recall that at a singular configuration there is at least one direction in which the endpoint cannot be moved at a non-zero velocity. But, if such a direction is not aligned with the positive y_0 direction, it is not a problem.

Before using the analytic form of the Jacobian, let us examine geometrically whether one of its column vectors can cover the positive y_0 direction at possible singular configurations.

Consider the semicircle periphery on the frontal plane, where the joint angles are:

$$\theta_1 = 0, \theta_3 = 0, 0 \le \theta_2 \le \pi$$

As shown in Figure S-6, the third column vector of the Jacobian can generate an endpoint velocity in the y_0 direction for all the peripheral configuration. This is also the case where $\theta_2 = \frac{\pi}{2}$. See Figure S-7. The column vector J_3 extends in the y direction.

In summary this design is fine except for the center line, the x_0 axis, that is a blind spot.

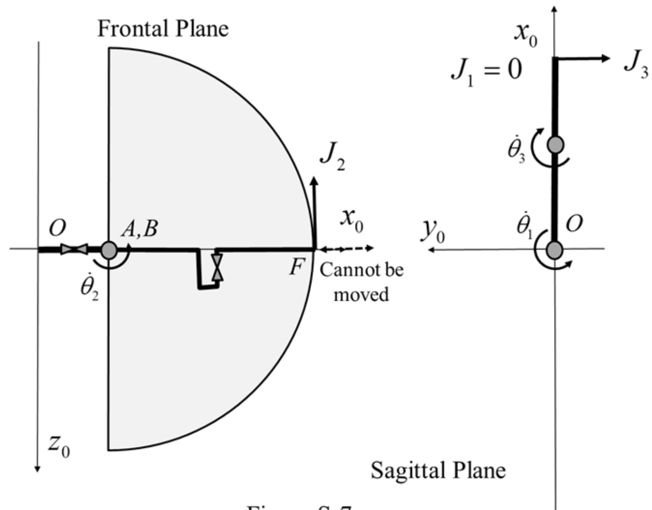


Figure S-7

3-e). The partial derivatives of the endpoint coordinates yield the Jacobian:

$$J = \begin{pmatrix} 0 & \ell c_2(1+c_3) & -\ell s_2 s_3 \\ \ell s_1 s_3 - \ell c_1 c_2(1+c_3) & \ell s_1 s_2(1+c_3) & -\ell(c_1 c_3 - s_1 c_2 s_3) \\ -\ell c_1 s_3 - \ell s_1 c_2(1+c_3) & -\ell c_1 s_2(1+c_3) & -\ell(s_1 c_3 + c_1 c_2 s_3) \end{pmatrix}$$

Solving $\det J = 0$, we can find where singularities occur. The determinant of the Jacobian can be reduced to: (Factor out ℓ 's and $(1+c_3)$ and expand with respect to the first column)

$$\det J = -c_2 s_3 (1+c_3)$$

Therefore, setting the determinant to zero, we find that the singularity configurations are:

i). $\cos\theta_2 = 0, \quad \theta_2 = \frac{\pi}{2}$, $J = \begin{pmatrix} 0 & 0 & -s_3 \\ s_1 s_3 & s_1(1+c_3) & -c_1 c_3 \\ -c_1 s_3 & -c_1(1+c_3) & -s_1 c_3 \end{pmatrix}$

ii). $\sin\theta_3 = 0, \quad \theta_3 = 0, \pi$, $J = \begin{pmatrix} 0 & 2c_2 & 0 \\ -2c_1 c_2 & 2s_1 s_2 & -c_1 \\ -2s_1 c_2 & -2c_1 s_2 & -s_1 \end{pmatrix}$

iii). $1+\cos\theta_3 = 0, \quad \theta_3 = -\frac{\pi}{2}$, $J = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & c_1 \\ 0 & 0 & s_1 \end{pmatrix}$

These Jacobian column vectors support the above arguments.

Point distribution:

2.12

Problem 1

a	10
b	10
c	10
d	10
e	10

Problem 2

a	10
b	10

Problem 3

a	10
b	10
c	10

Total 100

Extra Points

Problem 1 b*		5
Problem 2-G	a	10
	b	10
Problem 3	d	10
	e	10

2.120

Problem 1

a	10
b	10
c	10
d	10
e	10

Problem 2

a	10
b	10

Problem 2-G

a	10
b	10

Problem 3

a	10
b	10
c	10
d	10
e	10

Total 140

Extra Points

Problem 1	b*	5